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OPTIONS ON STOCK INDEXES, CURRENCIES, AND FUTURES

It turns out that options on stock indexes, currencies, and futures all have something in common. In each of those cases, the holder of the option does not get the same thing that the holder of the underlying asset gets. This affects the value of the option. In this note, we will cover the valuation implications of this for options on stock indexes, currencies, and futures.

Stock Indexes

One of the main issues for valuing options on stock indexes is dividends. While an individual stock might not pay dividends, for stock indexes it would be very unusual if at least one of the stocks included in the index was not paying a dividend. As of December 2003, there were options available on over 30 stock indexes.[1] Some of the more popular include: Dow Jones Industrial Average (DJX), NASDAQ 100 (NDX), Russell 2000 (RUT), and the S&P 500 (SPX). See **Exhibit 1** for some estimates of the number of securities included in each index, the basis on which the index is calculated,[2] the recent returns, volatilities, the percentage of dividend-paying stocks, and the average dividend yield. Note the similarities among the large cap indexes (Dow Jones and the S&P 500) and among the small cap indexes (Russell and the NASDAQ). In each case, the returns are similar as are the dividend yields. **Figure 1** shows the relative seven-year performance for 1996 through 2003 for each index. Note that the NASDAQ reflects the "dot.com" bubble of the late 1990's. **Figure 2** shows the time period 2000 to 2003. Without the bubble time period, the relative returns are much more similar.

[1] The CBOE offers options on a variety of indexes. See http://www.CBOE.com for a complete listing.

[2] In their most general form, indexes are simply an average value relative to some base. For example, we could construct an index with a base value of 100 today, and then from that point forward have a value index relative to 100. All the indexes listed above take this approach. All of them except for the Dow Jones Average, which is a price-weighted index, are value-weighted indexes. This means that the Dow Jones Average represents a simple average of prices and the others are market value-weighted averages. The major difference between them is that the Dow Jones gives more weight to the high-price stocks, while the others give more weight to the stocks with the highest market values.

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Figure 1. Relative performance of major indices (12/23/1996–12/23/2003).

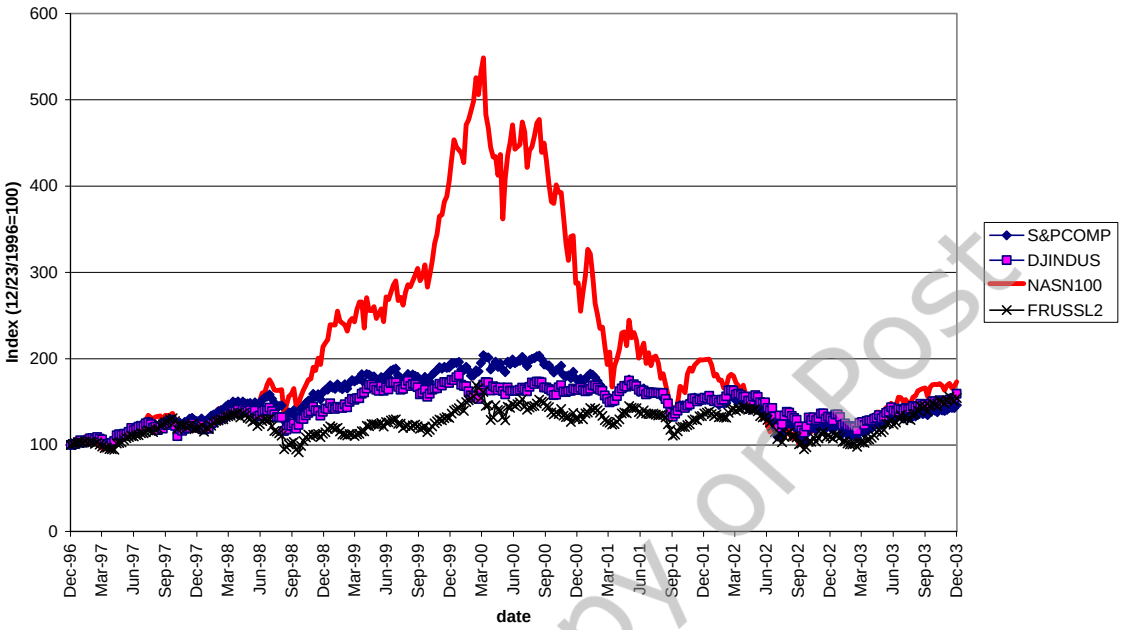
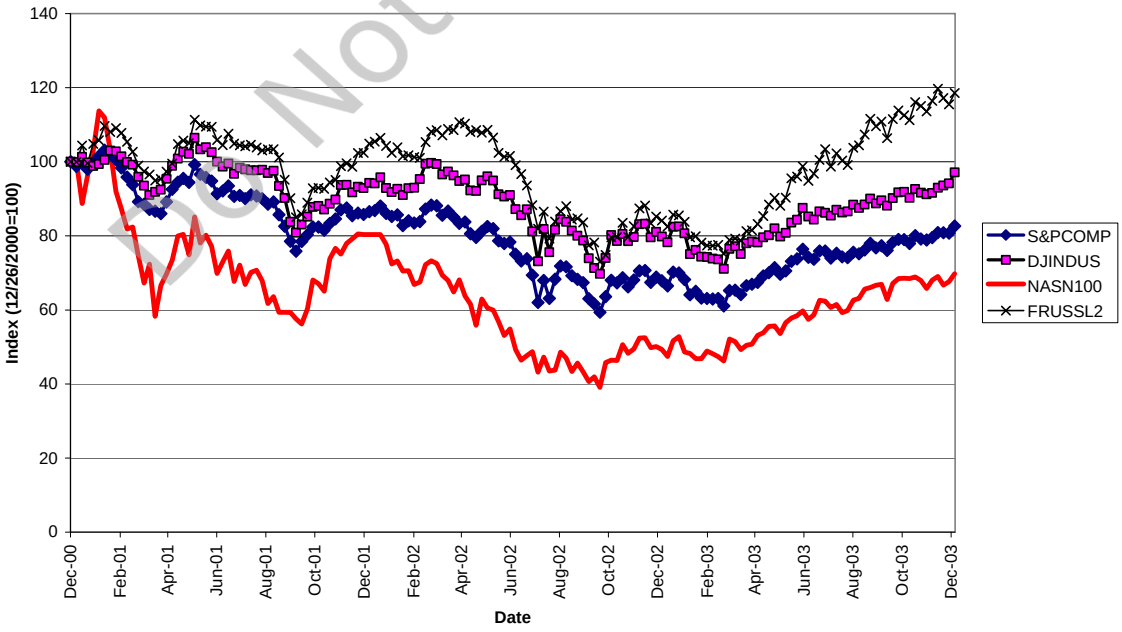


Figure 2. Relative performance of major indices (12/26/2000–12/23/2003).



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As we saw earlier[3] with options on individual stocks, we could account for the dividends by defining the underlying asset value (*UAV*) as the stock price less the present value of the dividends. This is relatively easy for options on individual stocks. However, when the number of stocks in an index is large, this is no longer a practical alternative. For options on stock indexes where the number of stocks in the index varies from 30 to 2,000, we almost always use a dividend yield approach to value options.

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Dividend Yield Approach to Value Options on Stock Indexes

We can use the Black-Scholes Model to value European call options on stock indexes. A typical call option on an index is the Chicago Board Options Exchange[4] (CBOE) S&P 500 Index option. See **Exhibit 2** for details on this option. Its payoff is

$$Payoff = Max\left[Index - Exercise, 0\right] \cdot \$100$$

It is a cash-settled option, which means that there is no buying or selling of the underlying asset, but rather the writer pays in cash to the holder upon exercise of the option at maturity. For example, suppose that the S&P 500 Index value[5] was 1,094, and we have a European call option with a maturity of three months and an exercise value of 1,100. The value of this European call option will depend on the exercise price (X = 1,100), the time to maturity (T = 3 months), the three-month risk-free rate[6] (r_f = 0.90%), volatility (σ = 0.21) and the underlying asset value.

As noted above, it is necessary to account for dividends in the underlying asset value. We do this by incorporating a dividend yield. In order to do this, we can use an approach similar to the one we used for the known dividend, which is to say, deduct the present value of the dividend from the stock price or index value.[7] Here, let S_0 stand as the current value of the index, R is the expected return on the index, T is the time to maturity on the option, and d_y is the dividend yield.

We can estimate the total dividend step by step. First, the index value should grow at a rate $R - d_y$. R is the total return and d_y is the dividend yield. The index value increases but at the same time it is growing, it is also paying out a dividend. As such, the net capital gains (index value) growth would be $R - d_y$ and the index value at time, T, would be

$$S_T = S_0 \cdot e^{(R-d_y)T}$$

[3] J. Hull, *Options, Futures and Other Derivatives*, 5th ed., (Upper Saddle River, NJ: Prentice Hall, 2002): 252–3.
[4] For information on the exchange, see http://www.cboe.com.
[5] See **Exhibit 1** for the values for the S&P 500 Index as of December 23, 2003.
[6] Three-month continuously compounded spot rate for U.S. government securities.
[7] The original work to price European call options on stocks with a dividend yield was first derived by R. Merton, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science* 4 (Spring 1973): 141–183.

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